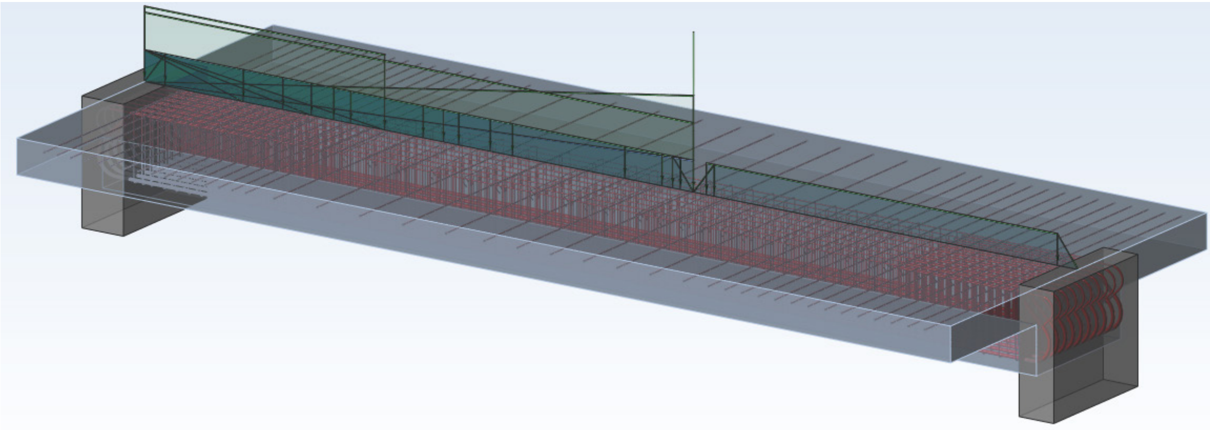
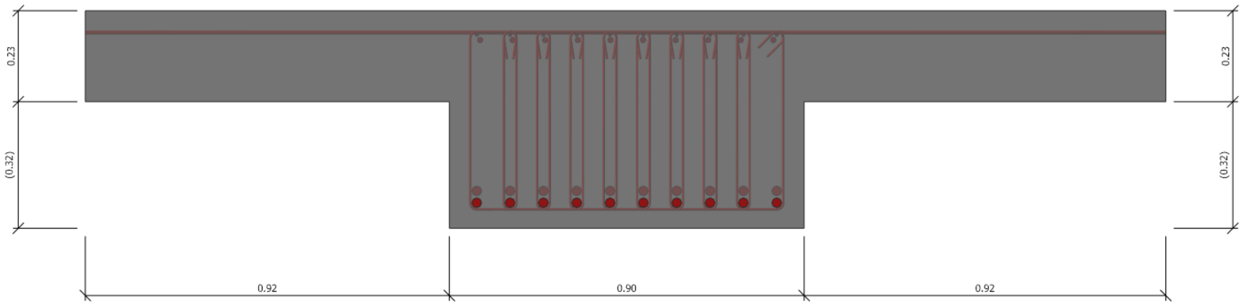
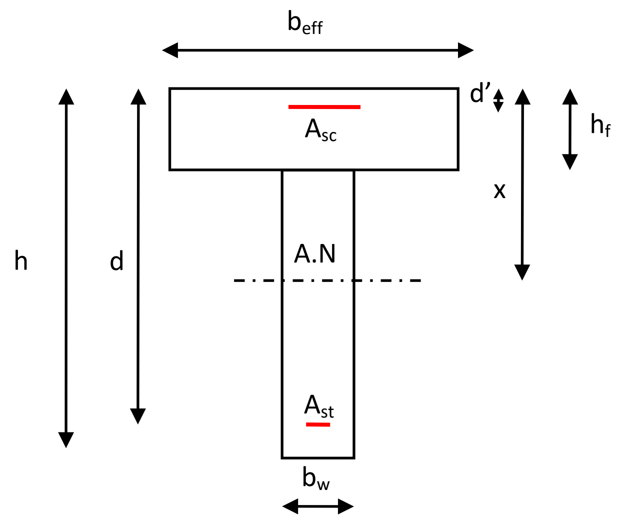
Peak deflection calculation (Professional Standards)





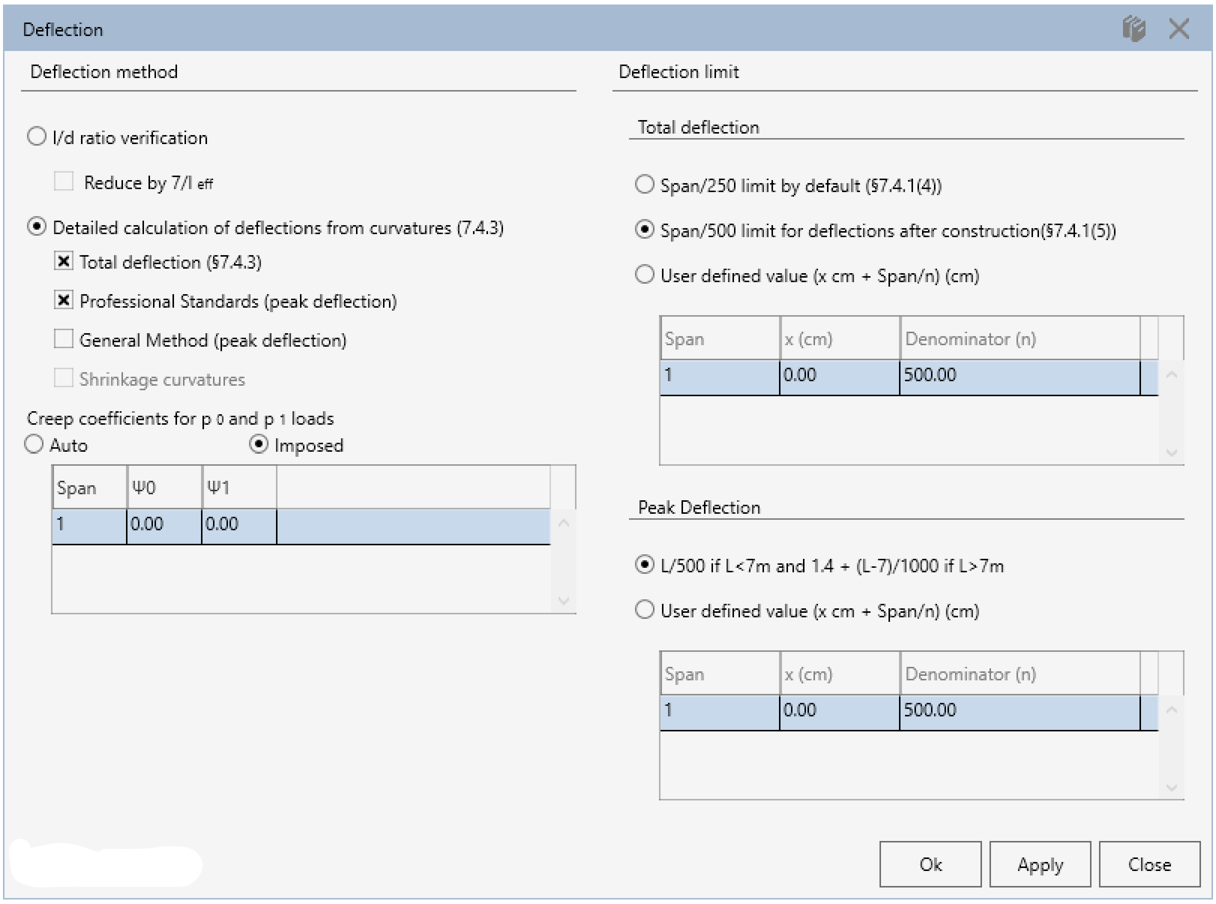
Calculation assumptions:

- h = 55 cm
- b_w = 90 cm
- d = 46.8 cm
- d’= 5.7 cm
- h_f = 23 cm
- b_{eff} = 274.1 cm



- A_{sc} = 0 cm^2 (assembly reinforcement; not taken into consideration in calculation)
- A_{st} = 98.174 cm^2
- Exposure class: XC1
- Materials:
  - ✓ f_{ck} = 40 MPa => f_{cm} = 48 MPa
  - ✓ f_{yk} = 500 MPa

=> Modular **ratio:** α_e = 15.



Estimation of uncracked moment of inertia

- **Equation of the neutral axis:**

$$x = \frac{\dfrac{b_w h^2}{2} + \dfrac{(b_{eff} - b_w)h_f^2}{2} + \alpha_e.(A_{st}.d + A_{sc}.d')}{b_w.h + (b_{eff} - b_w)h_f + \alpha_e.(A_{st} + A_{sc})}$$

x = 23.810 cm > h_f = 23 cm => The moment of inertia will be considered for the case of a T-section

- **The moment of inertia is equal to:**

$$I_h = \frac{b_{eff}.x^3}{3} - \frac{(b_{eff} - b_w)(x - h_f)^3}{3} + \frac{b_w(h - x)^3}{3} + \alpha_e.A_{st}.(d - x)^2 + \alpha_e.A_{sc}.(x - d')^2$$

I_h = 2921856.997 cm^4

Estimation of cracked moment of inertia

- **Equation of the neutral axis:**

$$\frac{b_w.x_1^2}{2} + \left[(b_{eff} - b_w)h_f + \alpha_e.(A_{st} + A_{sc})\right]x_1 - \left[(b_{eff} - b_w)\frac{h_f^2}{2} + \alpha_e.A_{st}.d + \alpha_e.A_{sc}.d'\right] = 0$$

By solving the 2nd degree equation:

x_1 = 18.040 cm < h_f = 23 cm => The moment of inertia will be considered for the case of a rectangular section (neutral axis in the flange)

- **The moment of inertia is equal to:**

$$I_e = \left[\frac{b_{eff}.x_1^3}{3} + A_{st}.\alpha_e.(d - x_1)^2 + A_{sc}.\alpha_e.(x_1 - d')^2\right]$$

I_e = 1754461.737 cm^4

Characteristics										
Span	α_e	ϕ	fctm,fl (MPa)	L (m)	E_v (MPa)	E_i (MPa)	Reinf. (cm^2)		Ih (m^4)	Ie_i (m^4)
							Abottom	Atop		
1	15.00	2.0	3.68	6.46	11740.15	35220.46	98.17	0.00	0.02922	0.01754

Total deflection calculation

- **Forces calculation**

Maximum bending moments					
Span	Mp (kN·m)	Mp+c (kN·m)	Mp+c+r (kN·m)	Mp+c+r+q (kN·m)	Mcr (kN·m)
1	70.17	70.17	1146.96	1289.88	345.14

Mcr *Critical moment*

- **Total deflection calculation**

For the total deflection calculation, we apply the (7.18) formula from EN1992-1-1, interpolating directly the deflection values:

$$f_t = \zeta . f_{et} + (1-\zeta) . f_{ht}$$

Where:

$$f_{et} = \frac{L^2}{10}\left[\frac{M_{p+c+r}}{E_{c,eff} . I_e} + \frac{M_q}{E_{cm} . I_e}\right]$$

$$f_{ht} = \frac{L^2}{10}\left[\frac{M_{p+c+r}}{E_{c,eff} . I_h} + \frac{M_q}{E_{cm} . I_h}\right]$$

The moments of inertia have been previously calculated:

- ✓ I_e : cracked moment of inertia => I_e = 1754461.737 cm^4
- ✓ I_h : uncracked moment of inertia => I_h = 2921856.997 cm^4

The instantaneous and effective Young's moduli for concrete are:

- ✓ Mean secant modulus of elasticity of concrete (short-term loading) =>

$$E_{cm} = 22000 . \left(\frac{f_{cm}}{10}\right)^{0,3} = 22000 . \left(\frac{48}{10}\right)^{0,3} = 35220.46 MPa$$

- ✓ Effective modulus of elasticity of concrete (long-term loading) =>

$$E_{c,eff} = \frac{35220.46}{3} = 11740.15 MPa$$

The interpolation coefficient ζ is determined as follows:

- ✓ If $M_{p+c+r+q} \geq M_{cr}$ => cracked moment of inertia calculation => $\zeta = 1 - \sqrt{\frac{M_{cr}}{M_{p+c+r+q}}}$.
- ✓ If $M_{p+c+r+q} < M_{cr}$ => uncracked moment of inertia calculation => $\zeta = 0$

The critical moment is equal to:

- ✓ $f_{ctm} = 0.30 f_{ck}^{2/3} = 0.30 \times 40^{2/3} = 3.51 MPa$
- ✓ $f_{ctm,fl} = \max\{(1,6 - h/1000).f_{ctm}; f_{ctm}\} = \max\{(1,6 - 550/1000).f_{ctm}; f_{ctm}\} = 3.68 MPa$
- ✓ $M_{cr} = \dfrac{f_{ctm,fl}.I_h}{(h-x)} = \dfrac{3.68 \times 0.029219}{(0.55 - 0.2381)} = 0.345136 MN.m = 345.1355 KN.m$

(where x corresponds to the section in uncracked state)

Maximum bending moments					
Span	Mp (kN·m)	Mp+c (kN·m)	Mp+c+r (kN·m)	Mp+c+r+q (kN·m)	Mcr (kN·m)
1	70.17	70.17	1146.96	1289.88	345.14

Mcr *Critical moment*

$$M_{p+c+r+q} = 1289.9 kNm \geq M_{cr} = 345.1 kNm$$

Thus, the interpolation coefficient is equal to $\zeta = 1 - \sqrt{\dfrac{M_{cr}}{M_{p+c+r+q}}} = 1 - \sqrt{\dfrac{345.1}{1289.9}} = 0.4827$

The total deflection calculation gives us:

L = 646 cm

M_{p+c+r} = 1147.0 kNm

M_q = 142.9 kNm

$$f_{et} = \frac{L^2}{10}\left[\frac{M_{p+c+r}}{E_{c,eff}.I_e} + \frac{M_q}{E_{cm}.I_e}\right] = \frac{6.46^2}{10}\left[\frac{1.147}{11740.15 \times 0.017619} + \frac{0.143}{35220.46 \times 0.017619}\right] = 0.0241 m = 2.42 cm$$

$$f_{ht} = \frac{L^2}{10}\left[\frac{M_{p+c+r}}{E_{c,eff}.I_h} + \frac{M_q}{E_{cm}.I_h}\right] = \frac{6.46^2}{10}\left[\frac{1.147}{11740.15 \times 0.029219} + \frac{0.143}{35220.46 \times 0.029219}\right] = 0.0145 m = 1.453 cm$$

$$f_t = \zeta.f_{et} + (1-\zeta).f_{ht} = 0.4827 \times 2.42 + (1 - 0.4827) \times 1.453 = 1.92 cm$$

Total deflection				
Span	ξt	Wet (cm)	Wht (cm)	Wt (cm)
1	0.48	-2.420	-1.453	-1.920

Wt = ξt · Wet + (1 - ξt) · Wht

Deflection EC2

Deflection EC2	
Calculated for 7.4.3 EC2	Uncracked Inertia Deflection -1.436 cm
	Cracked Inertia Deflection -2.511 cm
	Total deflection -2.334 cm
	Limit +/- 1.347 cm
Professional Method	Total deflection -1.920 cm
	Initial deflection -0.028 cm
	Peak deflection -1.892 cm
	Limit +/- 1.292 cm

Initial deflection calculation

The initial deflection is estimated starting from the following formula:

$$f_d = f_{dip} + \psi_0 (f_{dvp} - f_{dip})$$

The short-term calculation is then carried out:

$$f_{edip} = \frac{L^2}{10}\left[\frac{M_{p+c}}{E_{cm}.I_e}\right] = \frac{6.46^2}{10}\left[\frac{0.0702}{35220.46 \times 0.017619}\right] = 0.000472m = 0.047cm$$

$$f_{hdip} = \frac{L^2}{10}\left[\frac{M_{p+c}}{E_{cm}.I_h}\right] = \frac{6.46^2}{10}\left[\frac{0.0702}{35220.46 \times 0.029219}\right] = 0.000284m = 0.0284cm$$

$$f_{dip} = \zeta.f_{edip} + (1-\zeta).f_{hdip}$$

The interpolation coefficient ζ is determined as follows:

- If $M_{p+c} \geq M_{cr}$ => cracked inertia calculation => $\zeta = 1 - \sqrt{\frac{M_{cr}}{M_{p+c}}}$
- If $M_{p+c} < M_{cr}$ => non-cracked inertia calculation => $\zeta = 0$

M_{p+c} = 70.2 kNm < M_{cr} => $\zeta = 0$

$$f_{dip} = \zeta.f_{edip} + (1-\zeta).f_{hdip} = 0 \times 0.533 + (1-0) \times 0.0284 = 0.0284cm$$

Then the short-term/long-term calculation:

$$f_{edvp} = \frac{L^2}{10}\left[\frac{M_p}{E_{c,eff}.I_e} + \frac{M_c}{E_{cm}.I_e}\right] = \frac{6.46^2}{10}\left[\frac{0.0702}{11740.15 \times 0.017619} + \frac{0}{35220.46 \times 0.017619}\right] = 0.00141m = 0.142cm$$

$$f_{hdvp} = \frac{L^2}{10}\left[\frac{M_p}{E_{c,eff}.I_h} + \frac{M_c}{E_{cm}.I_h}\right] = \frac{6.46^2}{10}\left[\frac{0.0702}{11740.15 \times 0.029219} + \frac{0}{35220.46 \times 0.017619}\right] = 0.00085m = 0.0853cm$$

The interpolation coefficient ζ is determined as follows:

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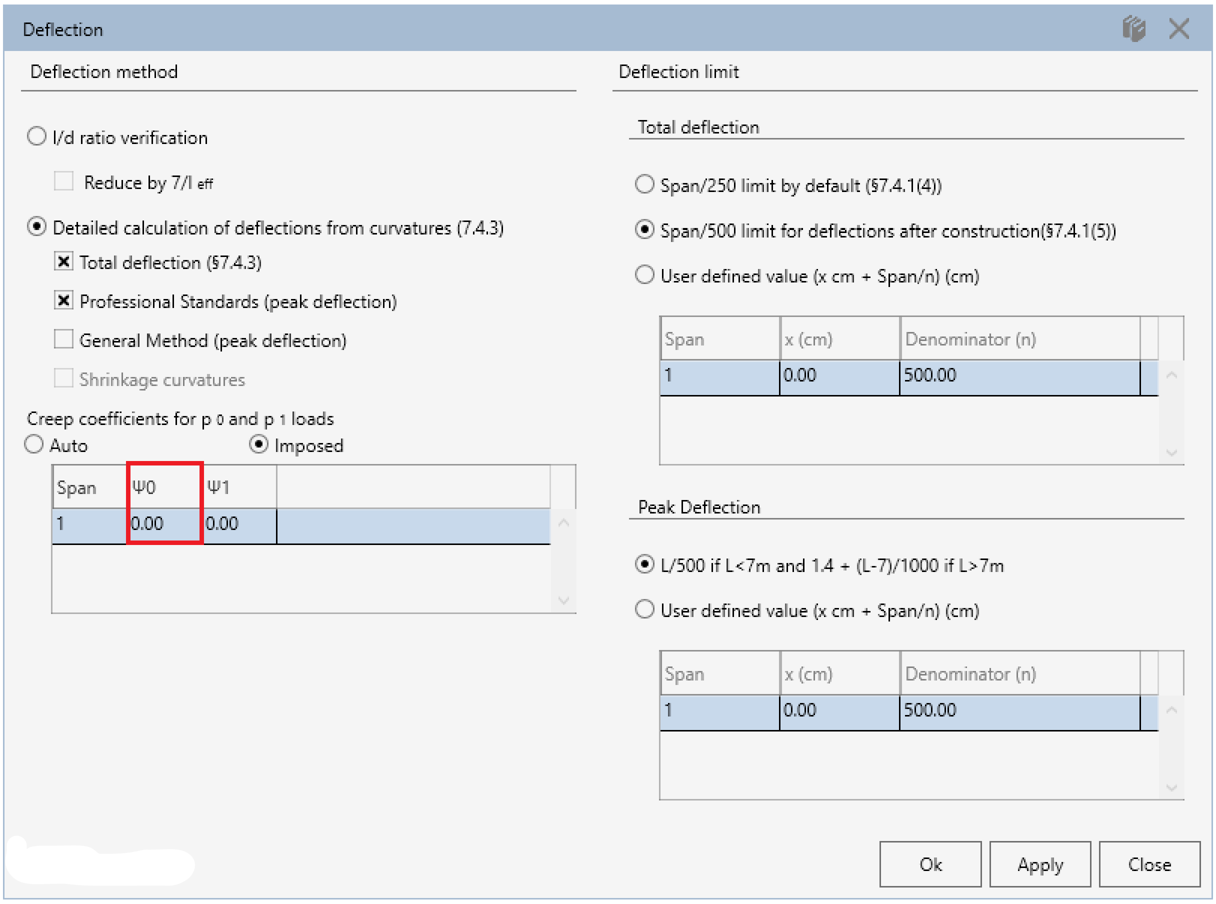
- If $M_{p+c} < M_{cr}$ => non-cracked inertia calculation => $\zeta = 0$

M_{p+c} = 70.2 kNm < M_{cr} => $\zeta = 0$

$$f_{dvp} = \zeta.f_{edvp} + (1-\zeta).f_{hdvp} = 0 \times 1.44 + (1-0) \times 0.085 = 0.0853cm$$



Initial deflection								
Span	RH (%)	h_0 (cm)	βH	tc (days)	t_0 (days)	$\beta c(tc;t_0)$	ψ	Wd (cm)
1	50	27.907	632.130	30	28	0.178	0.00	-0.03

The final value of the initial deflection is:

$$f_d = f_{dip} + \psi_0 (f_{dvp} - f_{dip}) = 0.0284 + 0.0(0.0853 - 0.0284) = 0.0284cm$$

Initial deflection					
Span		ξd	Wed (cm)	Whd (cm)	Wd (cm)
1	Short term	0.00	-0.047	-0.028	-0.028
	Long term	0.00	-0.142	-0.085	-0.085

Wd= ξd · Wed + (1 - ξd) · Whd

Deflection EC2

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	Limit +/- 1.292 cm

Peak deflection calculation

The peak deflection can be deduced directly from the two previously calculated deflection values:

$$f_{nu} = f_t - f_d = 1.92 - 0.0284 = 1.892cm$$

Peak deflection		
Span	**Wnu (cm)**	***f* max (cm)**
1	-1.892	1.292

Wd = Wdi + ψ · (Wdv - Wdi)

f max : *Permissible damaging deflection*

Wnu = Wt - Wd

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