Crack width verification

The crack opening calculation is done according to §7.3.4 (1) EN 1992-1-1.

The exposure class is XC1, which means that the maximum crack width is w_{max}= 0.4 mm

The calculation will be detailed for section at abscissa 0 mm, bottom position - $w_{k,bottom}$).

The value of the effective height (d) is defined automatically according to the real reinforcement in place.

Crack width verification										
Span - Section	Abscissa (mm)	Section Position	$w_{k,top}$ (mm)	$w_{k,bot}$ (mm)	$S_{r,max}$ (mm)	$\varepsilon_{sm}-\varepsilon_c$ (‰)	$w_{k,max}$ (mm)	w_{lim} (mm)	Work Ratio
1 - User section #1	0	Bottom	0.190	0.026	210	0.13	0.190	0.400	47.55 %

The crack width w_k may be calculated from the following expression:

$$w_k = s_{r,max} \times (\varepsilon_{sm} - \varepsilon_{cm})$$

$s_{r,max}$ is the maximum crack spacing

ε_{sm} is the mean strain in the reinforcement under the relevant combination of loads, including the effect of imposed deformations and taking into account the effects of tension stiffening.

Only the additional tensile strain beyond the state of zero strain of the concrete at the same level is considered.

ε_{cm} is the mean strain in the concrete between cracks

$\boldsymbol{\varepsilon_{sm} - \varepsilon_{cm}}$ calculation

$\varepsilon_{sm} - \varepsilon_{cm}$ may be calculated from the expression:

$$\varepsilon_{sm} - \varepsilon_{cm} = \max \begin{cases} \dfrac{\sigma_s - k_t \times \dfrac{fctm}{\rho_{p,eff}} \times (1 + \alpha_e \times \rho_{p,eff})}{E_s} \\ 0.6 \times \dfrac{\sigma_s}{E_s} \end{cases}$$

where:

σ_s is the stress in the tension reinforcement assuming a cracked section (the long term equivalence coefficient is used for calculation).

Short term equivalence coefficient

$$\alpha_e = \frac{E_s}{E_{cm}} = \frac{200000}{31475.81} = 6.35$$

Long term equivalence coefficient (used for neutral axis position, moment of inertia calculation and stresses calculation)

$$\alpha_e = \frac{\frac{E_s}{E_{cm}}}{1 + \phi(t, t_0) \times \frac{M_{Eqp}}{M_{Ecar}}}$$

Note: *The formula used for α_e depends on the option "Calculation of stresses according to professional recommendations" in Reinforced Concrete dialog, Cracking section.*

If the option is checked, the formula is the following:

$$\alpha_e = \frac{\frac{E_s}{E_{cm}}}{1 + \phi(t, t_0) \times \frac{M_{Eqp}}{M_{Ecar}}}$$

If the option is unchecked, the formula is the following:

$$\alpha_e = \frac{\frac{E_s}{E_{cm}}}{1 + \phi(t, t_0) \times \frac{M_{Eqp}}{M_{Ed}}}$$

where M_{Ed} is the bending moment in the specific combination for which the verification is done. It's either characteristic, frequent or quasi-permanent.

For $\phi(t, t_0)$ calculation, see the "Creep coefficient calculation" chapter.

$$\phi(t, t_0) = 2.56$$

$$E_{cm} = 22000 \frac{MN}{m^2} \times \left[\frac{f_{cm}}{10}\right]^{0.3} = 22000 \frac{MN}{m^2} \times \left[\frac{33}{10}\right]^{0.3} = 31475.81 \frac{MN}{m^2}$$

$$E_s = 200000 \frac{MN}{m^2}$$

Span - Section	Abscissa	Face	M_{Ed}	M_{cqc}	M_{fq}	M_{qp}	V_{Ed}	T_{Ed}
	(m)		(kN·m)	(kN·m)	(kN·m)	(kN·m)	(kN)	(kN·m)
1 - User section #1	0.00	Top	-190.48	-135.57	-110.71	-100.76	478.79	0.00
		Bottom	72.90	51.88	42.37	38.56	0.00	0.00

$$M_{Eqp} = 38.56 \text{ kN} \times \text{m}$$

$M_{Ecar} = 51.88 \, kN \times m$

$$\alpha_e = \frac{200000}{\frac{31475.81}{1 + 2.56 \times \frac{38.56}{51.88}}} = 18.40$$

$$\rho_{p,eff} = \frac{A_s}{A_{c,eff}}$$

$A_{c,eff}$ is the effective area of concrete in tension surrounding the reinforcement or prestressing tendons of depth, $h_{c,eff}$.

$$h_{c,eff} = \min \begin{cases} 2.5 \times (h - d) \\ \dfrac{h - x}{3} \\ \dfrac{h}{2} \end{cases}$$

Neutral axis position calculation

Neutral axis equation:

$$\frac{1}{2} \times b_w \times x^2 - A_{s,t} \times \alpha_e \times (d - x) + A_{s,c} \times \alpha_e \times (x - d') = 0$$

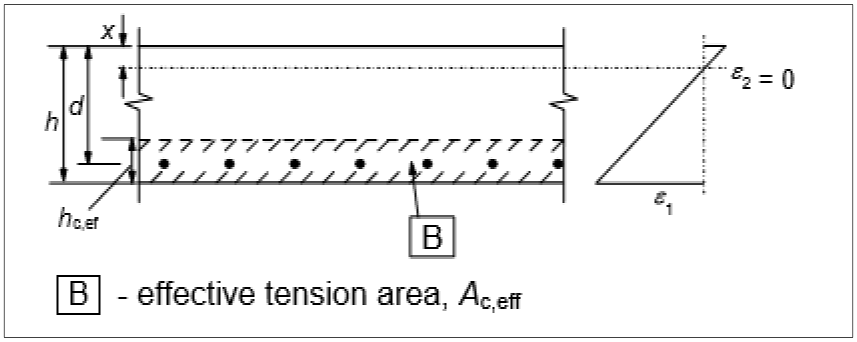
$$x = \frac{-\alpha_e \times \left(A_{s,c} + A_{s,t}\right) + \sqrt{\alpha_e^2 \times \left(A_{s,c} + A_{s,t}\right)^2 + 2 \times b_w \times \alpha_e \times (d' \times A_{s,c} + d \times A_{s,t})}}{b_w}$$

$$x = \frac{-18.40 \times (11.07 + 5.18) + \sqrt{18.40^2 \times (11.07 + 5.18)^2 + 2 \times 35 \times 18.44 \times (4.1 \times 5.18 + 90.6 \times 11.07)}}{35}$$

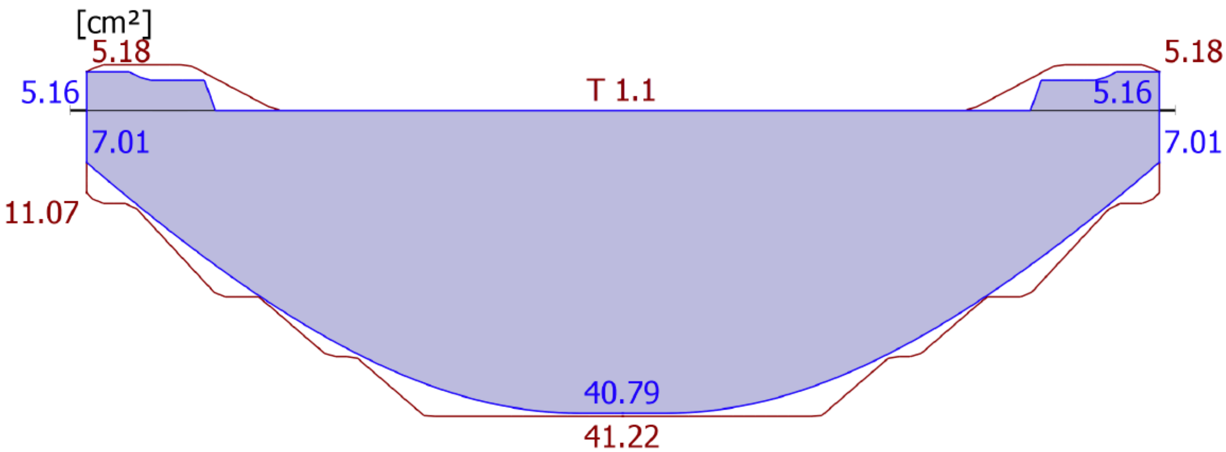
$x = 25.28 \, cm = 252.8 \, mm$

$$h_{c,eff} = \min \begin{cases} 2.5 \times (950 - 906) \\ \dfrac{950 - 252.8}{3} \\ \dfrac{950}{2} \end{cases}$$

$$h_{c,eff} = \min \begin{cases} 110 \, mm \\ 232.4 \, mm \\ 475 \, mm \end{cases} = 110 \, mm$$



$A_{c,eff} = b_w \times h_{c,eff} = 350 \times 110 \times 10^{-2} = 385\ cm^2$



According to the diagram above, the real longitudinal reinforcement value in top section $w_{k\ Max}$ (abscissa 0 mm) is:

$A_s = 11.07\ cm^2$

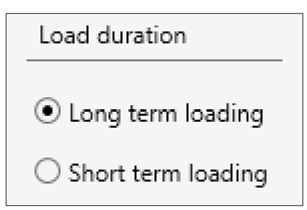
$$\rho_{p,eff} = \frac{A_s}{A_{c,eff}} = \frac{11.6}{385} = 0.03$$

k_t is a factor dependent on the duration of the load

k_t = 0,6 for short term loading

k_t = 0,4 for long term loading

The duration of the load can be defined in Reinforced concrete dialog, Load duration section:



Calculating the moment of inertia:

$$I = \left[\frac{b_w \times x^3}{3} + A_{s,c} \times \alpha_e \times (d' - x)^2 + A_{s,t} \times \alpha_e \times (d - x)^2\right]$$

$$= \left[\frac{35 \times 27.72^3}{3} + 11.6 \times 18.40 \times (90.6 - 25.28)^2 + \; 5.18 \times 18.40 \times (4.1 - 25.28)^2\right]$$

$$= 1100166.68 \; cm^4$$

Stresses calculation

$$\sigma_c = \frac{MEd}{I} \times x = \frac{38.56 \times 10^{-3}}{1100166.68 \; \times 10^{-8}} \times 0.2528 = 0.89 \frac{MN}{m^2}$$

$$\sigma_s = \alpha_e \times \sigma_c \times \frac{d - x}{x} = 18.40 \times 0.89 \times \frac{0.906 - 0.2528}{0.2528} = 42.12 \; \frac{MN}{m^2}$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \max\begin{cases}\dfrac{\sigma_s - k_t \times \dfrac{fctm}{\rho_{p,eff}} \times (1 + \alpha_e \times \rho_{p,eff})}{E_s} \\ 0.6 \times \dfrac{\sigma_s}{E_s}\end{cases}$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \max\begin{cases}\dfrac{42.12 - 0.4 \times \dfrac{2.56}{0.03} \times (1 + 18.40 \times 0.03)}{200000} \\ 0.6 \times \dfrac{42.12}{200000}\end{cases} = 0.000126$$

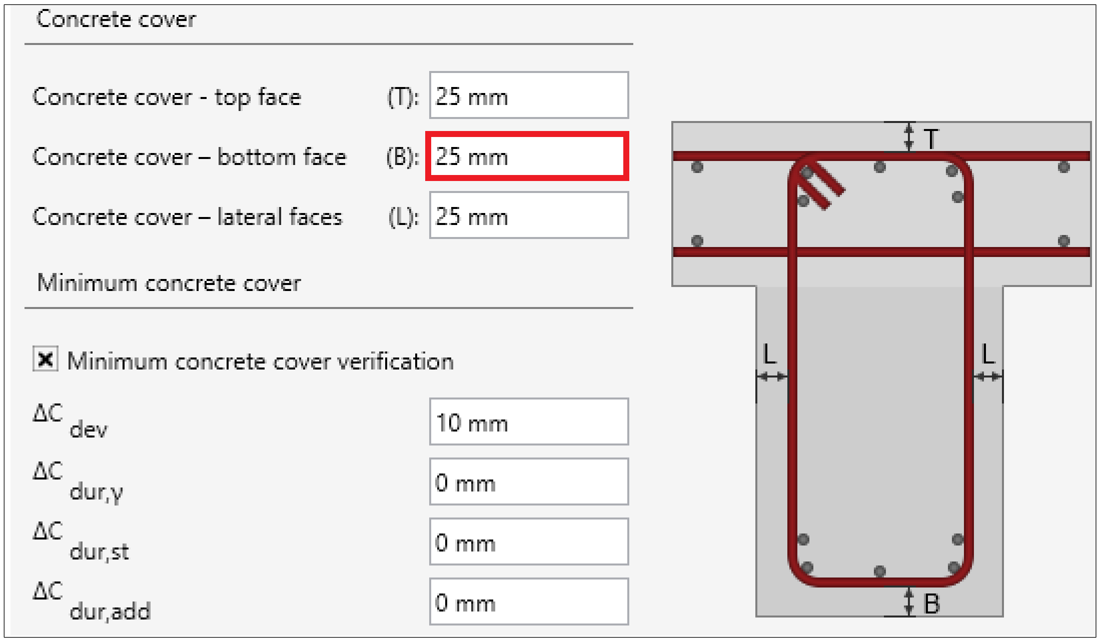
$s_{r,max}$ calculation

$$s_{r,max} = k_3 \times c + k_1 \times k_2 \times k_4 \times \frac{\phi}{\rho_{p,eff}}$$

ϕ is the bar diameter. Where a mixture of bar diameters is used in a section, an equivalent diameter, ϕ_{eq}, should be used. For a section with n_1 bars of diameter ϕ_1 and n_2 bars of diameter ϕ_2, the following expression should be used:

$$\phi_{eq} = \frac{n_1 \times \phi_1^2 + n_2 \times \phi_2^2}{n_1 \times \phi_1 + n_2 \times \phi_2}$$

c is the cover to the longitudinal reinforcement



$c = B + \phi_{transversal} = 25 + 6 = 31$ mm where:

B is the bottom cover of the link and can be defined in Concrete covers dialog.

k_1 is a coefficient which takes account of the bond properties of the bonded reinforcement:

= 0,8 for high bond bars

= 1,6 for bars with an effectively plain surface (e.g. prestressing tendons)

k_2 is a coefficient which takes account of the distribution of strain:

= 0,5 for bending

= 1,0 for pure tension

Note: *The values of k_3 and k_4 for use in a Country may be found in its National Annex. The recommended values are 3,4 and 0,425 respectively.*

For France, there is a discussion for k_3:

If $c \geq 25$ mm, $k_3 = 3.4 \times \left(\frac{25}{c}\right)^{\frac{2}{3}}$ otherwise $k_3 = 3.4$

For the rest of the countries, RC Beam Designer considers $k_3 = 3.4$.

In this example, $c = 31$ mm => $k_3 = 3.4 \times \left(\frac{25}{c}\right)^{\frac{2}{3}} = 3.4 \times \left(\frac{25}{31}\right)^{\frac{2}{3}} = 2.95$

$$s_{r,max} = k_3 \times c + k_1 \times k_2 \times k_4 \times \frac{\phi}{\rho_{p,eff}}$$

$$s_{r,max} = 2.95 \times 31 + 0.8 \times 0.5 \times 0.425 \times \frac{20}{0.03}$$

$s_{r,max} = 209.57\ \text{mm}$

w_k calculation

$w_k = s_{r,max} \times (\varepsilon_{sm} - \varepsilon_{cm}) = 209.57 \times 0.000126 = 0.026\ \text{mm} \leq w_{max} = 0.4\ \text{mm}$ Satisfied

The value for w_{max} can be defined in Reinforced Concrete dialog,Cracking section.

The user has two options:

- By default when selecting the exposure class the maximum crack width limit updates accordingly; this value will be taken into account in calculation.
- If the option *Imposed crack width* is checked, the user is allowed to manually input a value for the maximum crack width limit which will be further considered in calculation.