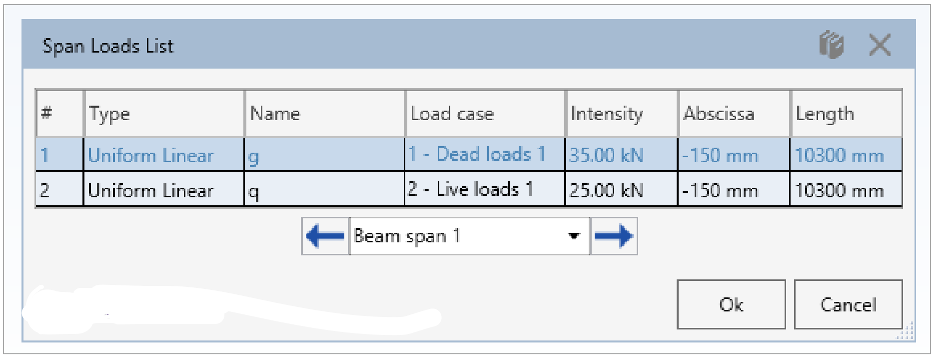
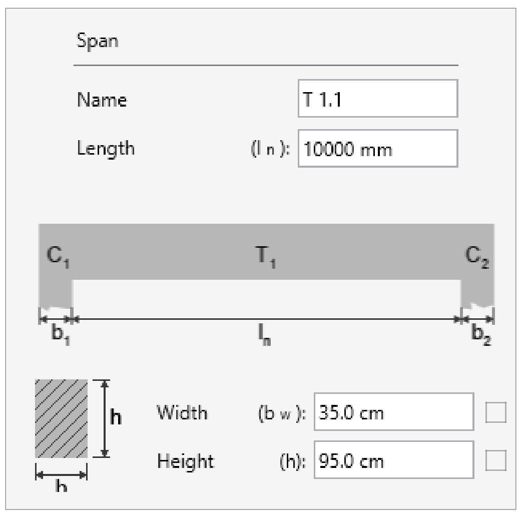
Longitudinal reinforcement

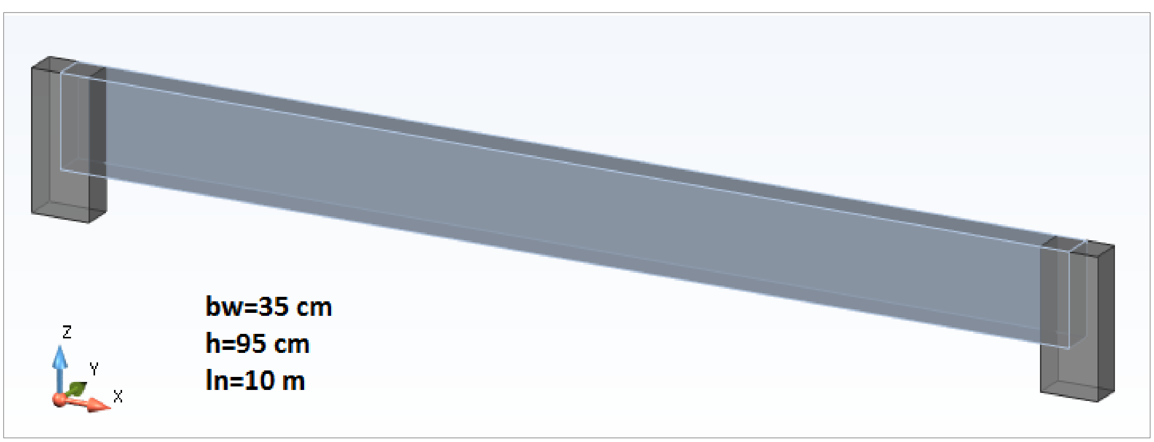
Example 1

The beam considered has a rectangular section of 0.35x0.95 m and a length of 10 m. The concrete class is 25/30 and the steel is S500.The loads on beam are the following:

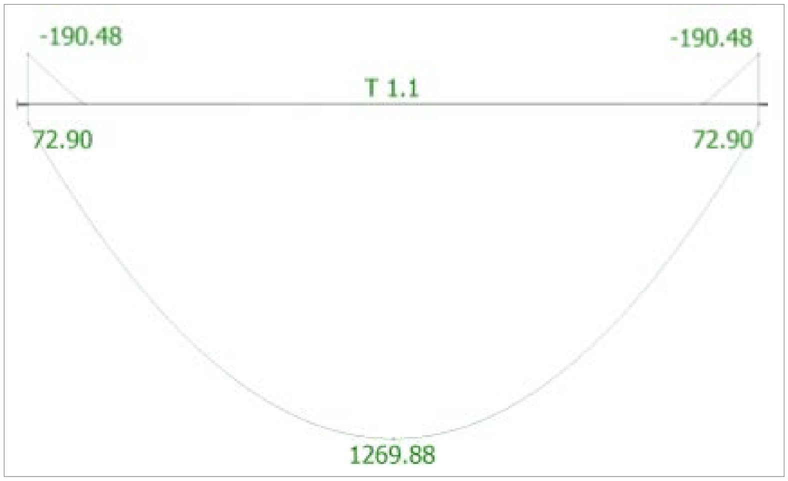


#	Type	Name	Load case	Intensity	Abscissa	Length
1	Uniform Linear	g	1 - Dead loads 1	35.00 kN	-150 mm	10300 mm
2	Uniform Linear	q	2 - Live loads 1	25.00 kN	-150 mm	10300 mm





The ULS bending moment envelope is the following:



Synthetic values table:

Span	Section	Abscissa	Position	M_{Ed}	M_{cqc}	M_{fq}	M_{qp}	V_{Ed}	T_{Ed}
		(mm)		(kN·m)	(kN·m)	(kN·m)	(kN·m)	(kN)	(kN·m)
1	Left Support	0	Top	-190.48	-135.57	-110.71	-100.76	478.79	0.00
			Bottom	72.90	51.88	42.37	38.56	0.00	0.00
1	Right Support	10000	Top	-190.48	-135.57	-110.71	-100.76	0.00	0.00
			Bottom	72.90	51.88	42.37	38.56	-478.79	0.00
1	MInf	5000	Top	0.00	0.00	0.00	0.00	0.00	0.00
			Bottom	1269.88	903.81	738.05	671.74	0.00	0.00
1	VInf	10000	Top	-190.48	-135.57	-110.71	-100.76	0.00	0.00
			Bottom	72.90	51.88	42.37	38.56	-478.79	0.00

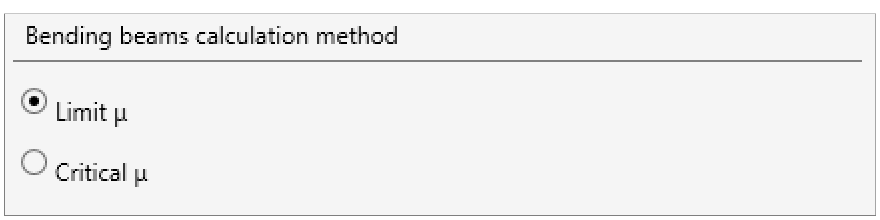
<u>Longitudinal reinforcement calculation</u>

The longitudinal reinforcement calculation will be detailed for two methods:

- Limit reduced moment method
- Critical reduced moment

The calculation of the limit reduced moment is necessary for compressed reinforcement area calculation.

The RC Beam Designer offers two options to determine the limit reduced moment:



Limit reduced moment corresponds only to the criterion of steel elongation which should be less than ε_{se} corresponding to the elastic limit.

Critical reduced moment method considers the fact that the compression limit stress should not be exceeded.

According to the EC2, the choice of limit reduced moment method depends on the exposure class.

For exposure classes X0-XA-XC, no checking for the compression limit stress of the concrete is necessary, and the method "μ limit" can be used. For other exposure classes, the compression stress in the concrete should be limited at $0.6 \text{ x } f_{ck}$, so it is preferable to apply the method "μ critical".

In the RC Beam Designer, the choice is left to the user and the software returns a warning message in case the exposure class exceeds XC and the method chosen is "μ limit ".

- ✓ **Limit μ method**

The calculation will be detailed for section M_{inf} (abscissa 5000 mm) resulting bottom bars reinforcement area and for section Left/Right Support (abscissa 0/10000 mm) resulting top bars reinforcement area.

The value of the effective height (d) is defined automatically, according to the real reinforcement in place.

<u>Bottom bars</u>

Design bending moment $M_{Ed} = -1269.88 \text{ kN} \times \text{m}$

Reduced moment:

$$\mu_{cu} = \frac{M_{Ed}}{b_w \times d^2 \times fcd} = \frac{1269.88 \times 10^{-3}}{0.35 \times 0.868^2 \times 16.67} = 0.29$$

Neutral axis position:

$$x_r = \frac{1}{\lambda} \times \left(1 - \sqrt{1 - 2 \times \mu_{cu}}\right) = \frac{1}{0.8} \times \left(1 - \sqrt{1 - 2 \times 0.29}\right) = 0.44$$

Inner lever arm:

$$z_b = d \times (1 - 0.5 \times \lambda \times x_r) = 868 \times (1 - 0.5 \times 0.8 \times 0.44) = 715.2 \text{ mm} \sim 716 \text{ mm}$$

$$A_{sl} = \frac{M_{Ed}}{z_b \times fyd} = \frac{1269.88 \times 10^{-3}}{0.716 \times 434.78} \times 10^4 = 40.79 \text{ cm}^2$$

<u>Top bars</u>

Design bending moment $M_{Ed} = 190.48 \text{ kN} \times \text{m}$

Reduced moment:

$$\mu_{cu} = \frac{M_{Ed}}{b_w \times d^2 \times fcd} = \frac{190.48 \times 10^{-3}}{0.35 \times 0.868^2 \times 16.67} = 0.04$$

Neutral axis position:

$$x_r = \frac{1}{\lambda} \times \left(1 - \sqrt{1 - 2 \times \mu_{cu}}\right) = \frac{1}{0.8} \times \left(1 - \sqrt{1 - 2 \times 0.04}\right) = 0.05$$

Inner lever arm:

$$z_b = d \times (1 - 0.5 \times \lambda \times x_r) = 868 \times (1 - 0.5 \times 0.8 \times 0.05) = 850.6 \text{ mm} \sim 851 \text{ mm}$$

$$A_{sl} = \frac{M_{Ed}}{z_b \times fyd} = \frac{190.48 \times 10^{-3}}{0.851 \times 434.78} \times 10^4 = 5.15 \text{ cm}^2$$

The minimal and maximal percentages of longitudinal reinforcement in the beam are defined by the article §9.2.1.1 from EN1992-1-1:

Minimal percentage:

$$A_{s,min} = \max \begin{cases} 0.26 \times \dfrac{fctm}{fyk} \times b_w \times d \\ 0.0013 \times b_w \times d \end{cases}$$

$$A_{s,min} = \max \begin{cases} 0.26 \times \dfrac{2.56}{500} \times 0.35 \times 0.868 \times 10^4 \\ 0.0013 \times 0.35 \times 0.868 \times 10^4 \end{cases}$$

$$A_{s,min} = \max \begin{cases} 4.05 \text{ cm}^2 \\ 3.95 \text{ cm}^2 \end{cases} = 4.05 \text{ cm}^2$$

Maximal percentage:

$$A_{s,max} = 0.04 \times A_c = 0.04 \times 0.35 \times 0.95 \times 10^4 = 133 \text{ cm}^2$$

✓ **Critical μ method**

The calculation will be detailed for section M_{inf} (abscissa 5000 mm) resulting bottom bars reinforcement area and for section Left/Right Support (abscissa 0/10000 mm) resulting top bars reinforcement area.

The value of the effective height (d) is defined automatically according the real reinforcement in place.

<u>Bottom bars</u>

Design bending moment $M_{Ed} = -1269.88 \text{ kN} \times \text{m}$

Reduced moment:

$$\mu_{cu} = \frac{M_{Ed}}{b_w \times d^2 \times fcd} = \frac{1269.88 \times 10^{-3}}{0.35 \times 0.868^2 \times 16.67} = 0.29$$

Reduced moment limit, calculated using the formula defined by Jeans Roux in his book "Practice of EC2, formulas available for steel S500 and $f_{ck} \leq 50$ MPa:

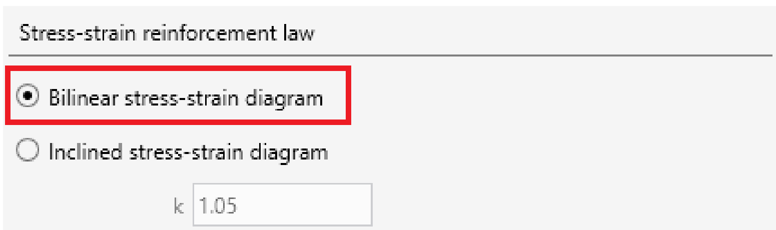
- for bilinear stress-strain diagram(with horizontal top branch)

$$\mu_{lim} = \frac{f_{ck}}{(4.69 - 1.7 \times \gamma) \times f_{ck} + (159.90 - 76.20 \times \gamma)} \times K$$

- for inclined stress-strain diagram

$$\mu_{lim} = \frac{f_{ck}}{(4.62 - 1.66 \times \gamma) \times f_{ck} + (165.69 - 79.62 \times \gamma)} \times K$$

In this example the option Bilinear stress-strain diagram is checked in the Reinforced Concrete dialog:



K is a factor which takes into account the equivalent coefficient α_e:

$$K(\alpha_e) = (A + B \times \alpha_e + C \times \alpha_e^2) \times 10^{-4}$$

- for bilinear stress-strain diagram(with horizontal top branch)

  $A = 71.2 \times f_{ck} + 108 = 71.2 \times 25 + 108 = 1888$

  $B = -5.2 \times f_{ck} + 847.4 = -5.2 \times 25 + 847.4 = 717.4$

  $C = 0.03 \times f_{ck} - 12.5 = 0.03 \times 25 - 12.5 = -11.75$

- for inclined stress-strain diagram

  $A = 75.3 \times f_{ck} - 189.8$

  $B = -5.6 \times f_{ck} + 874.5$

  $C = 0.04 \times f_{ck} - 13$

Equivalent coefficient $\boldsymbol{\alpha_e}$ calculation

To determine the equivalence coefficient, we must first estimate the creep coefficient, related to elastic deformation at 28 days and 50% humidity:

$$\alpha_e = \frac{\frac{E_s}{E_{cm}}}{1 + \phi(t, t_0) \times \frac{M_{Eqp}}{M_{Ecar}}}$$

For ϕ(t,t0) calculation, see the "Creep coefficient calculation" chapter.

$$\phi(t, t_0) = 2.56$$

$$E_{cm} = 22000 \frac{MN}{m^2} \times \left[\frac{f_{cm}}{10}\right]^{0.3} = 22000 \frac{MN}{m^2} \times \left[\frac{33}{10}\right]^{0.3} = 31475.81 \frac{MN}{m^2}$$

$$E_s = 200000 \frac{MN}{m^2}$$

$$M_{Eqp} = 671.74\ kN$$

$$M_{Ecar} = 903.81\ kN$$

$$\alpha_e = \frac{200000}{\frac{31475.81}{1 + 2.56 \times \frac{671.74}{903.81}}} = 18.44$$

$$K(\alpha_e) = (A + B \times \alpha_e + C \times \alpha_e^2) \times 10^{-4} = (1888 + 717.4 \times 18.44 - 11.75 \times 18.44^2) \times 10^{-4} = 1.11$$

$$\gamma = \frac{M_{Ed}}{M_{SLS}} = \frac{1269.88}{903.81} = 1.41$$

$$\mu_{lim} = \frac{f_{ck}}{(4.69 - 1.7 \times \gamma) \times f_{ck} + (159.90 - 76.20 \times \gamma)} \times K$$

$$\mu_{lim} = \frac{25}{(4.69 - 1.7 \times 1.41) \times 25 + (159.90 - 76.20 \times 1.41)} \times 1.11$$

$$\mu_{lim} = 0.25$$

$\mu_{cu} = 0.29 > \mu_{lim} = 0.25 =>$ compressed reinforcement area is required

Neutral axis position:

$$x_r = \frac{1}{\lambda} \times \left(1 - \sqrt{1 - 2 \times \mu_{lim}}\right) = \frac{1}{0.8} \times \left(1 - \sqrt{1 - 2 \times 0.025}\right) = 0.37$$

Inner lever arm:

$$z_b = d \times (1 - 0.5 \times \lambda \times x_r) = 868 \times (1 - 0.5 \times 0.8 \times 0.37) = 739.54\ mm$$

Critical moment:

$$M_{cr} = \mu_{lim} \times b_w \times d^2 \times f_{cd} = 0.25 \times 0.35 \times 0.868^2 \times 16.67 \times 10^3 = 1098.96\ kN \times m$$

Compressed reinforcement area:

$$A_{s2} = \frac{M_{Ed} - M_{cr}}{\sigma_{sc} \times (d - d')} = \frac{(1269.88 - 1098.96) \times 10^{-3}}{434.78 \times (0.868 - 0.025)} \times 10^4 = 4.66\ cm^2$$

Tension reinforcement area:

$$A_{s1} = \frac{M_{cr}}{z_b \times \sigma_s} + A_{s2} \times \frac{\sigma_{sc}}{\sigma_s} = \left(\frac{1098.96 \times 10^{-3}}{0.739 \times 434.78}\right) \times 10^4 + 4.66 \times \frac{434.78}{434.78} = 38.86\ cm^2$$

<u>Top bars</u>

Design bending moment $M_{Ed} = 190.48\ kN \times m$

Reduced moment:

$$\mu_{cu} = \frac{M_{Ed}}{b_w \times d^2 \times fcd} = \frac{190.48 \times 10^{-3}}{0.35 \times 0.868^2 \times 16.67} = 0.04$$

Reduced moment limit, calculated using the formula defined by Jeans Roux in his book “Practice of EC2, formulas available for steel S500 and $f_{ck} \leq 50\ \text{MPa}$:

- for bilinear stress-strain diagram(with horizontal top branch)

$$\mu_{lim} = \frac{f_{ck}}{(4.69 - 1.7 \times \gamma) \times f_{ck} + (159.90 - 76.20 \times \gamma)} \times K$$

$$K(\alpha_e) = (A + B \times \alpha_e + C \times \alpha_e^2) \times 10^{-4}$$

- for bilinear stress-strain diagram(with horizontal top branch)

$$A = 71.2 \times f_{ck} + 108 = 71.2 \times 25 + 108 = 1888$$

$$B = -5.2 \times f_{ck} + 847.4 = -5.2 \times 25 + 847.4 = 717.4$$

$$C = 0.03 \times f_{ck} - 12.5 = 0.03 \times 25 - 12.5 = -11.75$$

Equivalent coefficient $\boldsymbol{\alpha_e}$ calculation

To determine the equivalence coefficient, we must first estimate the creep coefficient, related to elastic deformation at 28 days and 50% humidity:

$$\alpha_e = \frac{\frac{E_s}{E_{cm}}}{1 + \phi(t, t_0) \times \frac{M_{Eqp}}{M_{Ecar}}}$$

For $\phi(t,t0)$ calculation, see the ”Creep coefficient calculation” chapter.

$$\phi(t, t_0) = 2.56$$

$$E_{cm} = 22000 \frac{MN}{m^2} \times \left[\frac{f_{cm}}{10}\right]^{0.3} = 22000 \frac{MN}{m^2} \times \left[\frac{33}{10}\right]^{0.3} = 31475.81 \frac{MN}{m^2}$$

$$E_s = 200000 \frac{MN}{m^2}$$

$$M_{Eqp} = 100.76\ \text{kN}$$

$$M_{Ecar} = 135.57\ \text{kN}$$

$$\alpha_e = \frac{\frac{200000}{31475.81}}{1 + 2.56 \times \frac{100.76}{135.57}} = 18.44$$

$$K(\alpha_e) = (A + B \times \alpha_e + C \times \alpha_e^2) \times 10^{-4} = (1888 + 717.4 \times 18.44 - 11.75 \times 18.44^2) \times 10^{-4} = 1.11$$

$$\gamma = \frac{M_{Ed}}{M_{SLS}} = \frac{190.48}{135.57} = 1.41$$

$$\mu_{lim} = \frac{f_{ck}}{(4.69 - 1.7 \times \gamma) \times f_{ck} + (159.90 - 76.20 \times \gamma)} \times K$$

$$\mu_{lim} = \frac{25}{(4.69 - 1.7 \times 1.41) \times 25 + (159.90 - 76.20 \times 1.41)} \times 1.11$$

$$\mu_{lim} = 0.25$$

$\mu_{cu} = 0.04 < \mu_{lim} = 0.25$ => compressed reinforcement area is not required

Neutral axis position:

$$x_r = \frac{1}{\lambda} \times \left(1 - \sqrt{1 - 2 \times \mu_{cu}}\right) = \frac{1}{0.8} \times \left(1 - \sqrt{1 - 2 \times 0.04}\right) = 0.05$$

Inner lever arm:

$$z_b = d \times (1 - 0.5 \times \lambda \times x_r) = 868 \times (1 - 0.5 \times 0.8 \times 0.05) = 850.6 \text{ mm} \sim 851 \text{ mm}$$

$$A_{sl} = \frac{M_{Ed}}{z_b \times fyd} = \frac{190.48 \times 10^{-3}}{0.851 \times 434.78} \times 10^4 = 5.15 \text{ cm}^2$$

Minimal percentage:

$$A_{s,min} = \max \begin{cases} 0.26 \times \dfrac{fctm}{fyk} \times b_w \times d \\ 0.0013 \times b_w \times d \end{cases}$$

$$A_{s,min} = \max \begin{cases} 0.26 \times \dfrac{2.56}{500} \times 0.35 \times 0.868 \times 10^4 \\ 0.0013 \times 0.35 \times 0.868 \times 10^4 \end{cases}$$

$$A_{s,min} = \max \begin{cases} 4.05 \text{ cm}^2 \\ 3.95 \text{ cm}^2 \end{cases} = 4.05 \text{ cm}^2$$

Maximal percentage:

$$A_{s,max} = 0.04 \times A_c = 0.04 \times 0.35 \times 0.95 \times 10^4 = 133 \text{ cm}^2$$