Response spectrum curves (ASCE)

One of the central problems of engineering seismology is the calculation of the behavior of a structure subjected to a given ground motion. One attempt to simplify this problem has involved the introduction of so-called “response spectrum”.

For the practical seismic design of structures, simplified response spectra that represent the hazard of a site are used. These spectra represent average values from a number of possible earthquakes scenarios that could affect the site under consideration.

These values which were derived by studying the spectral values of a large number of earthquake records represent the average relation between different ground seismic parameters. For a given site, they are scaled directly to the maximum design acceleration, which is a function of the seismic hazard of the region. The simplified spectrum is obtained by multiplying each branch of the ground parameters by an amplification factor that depends on the damping coefficient of the structure and the required probability of exceedance. For the seismic design of structures of normal importance, it was suggested that the values corresponding to a probability of exceedance of 50% could be used.

Based on these concepts, design codes define design response spectra for all the different regions in their territory of application.

The design response spectrum is used to determine the design spectral response accelerations for a given structure. After calculating design response acceleration coefficients $S_{DS}$ and $S_{DL}$, the design response spectrum curve should be constructed as it follows:

In USA, the probabilistic approach is treated. For each specific site, you define a Maximum Considered Earthquake (MCE) (an event with a 2% probability of exceedance in 50 years or a $T_r = 2475$ years). The design earthquake is $2/3$ the MCE.

Simplified design response spectrum according to ASCE 7-10
• For periods less than $T_0$, the design spectral response acceleration, $S_a$, shall be taken as given by Eq. 11.4-5:

$$S_a = S_{DS} \left(0.4 + 0.6 \frac{T}{T_0}\right)$$

• For periods greater than or equal to $T_0$ and less than or equal to $T_S$, the design spectral response acceleration, $S_a$, shall be taken equal to $S_{DS}$.

[Diagram showing the spectral response acceleration for different periods]
For periods greater than $T_S$, and less than or equal to $T_L$, the design spectral response acceleration, $S_a$, shall be taken as given by Eq. 11.4-6:

$$S_a = \frac{S_{D1}}{T}$$

Where:

$S_{D1}$ = the design spectral response acceleration parameter at 1-s period

For periods greater than $T_L$, $S_a$ shall be taken as given by Eq. 11.4-7:

$$S_a = \frac{S_{D1}T_L}{T^2}$$

Where:

$S_{D1}$ = the design spectral response acceleration parameter at 1-s period
Earthquake ground motion is usually recorded as an acceleration of the ground at a particular location. The acceleration of the ground generates the acceleration of the structure (response acceleration), which produces earthquake forces that act on the structure. Earthquake forces generate deformations, internal forces and stresses in the structure. Therefore, the first step to design an earthquake-resistant structure is to determine the maximum possible response acceleration that can occur during the earthquake. It is also important to know that response of the given structure depends on the period of vibration and damping characteristics of the structure.

The ASCE 7-10, Section 11.4.5 describes the procedure to determine the design response spectrum curve, from which the design response accelerations, $S_a$, for any given period of vibration $T$ are calculated. One part of this procedure is the determination of the spectral response acceleration coefficients for short periods, $S_{DS}$, and for a 1-second period, $S_{D1}$.

Design earthquake spectral response acceleration parameter at short period, $S_{DS}$, and at 1 s period, $S_{D1}$, shall be determined from Eqs. 11.4-3 and 11.4-4, respectively.

\begin{align*}
S_{DS} &= \frac{2}{3} S_{MS} \\
S_{D1} &= \frac{2}{3} S_{M1}
\end{align*}

Where:

$S_{MS}$ = the MCER, 5 percent damped, spectral response acceleration parameter at short periods adjusted for site class effects as defined in Section 11.4.3

$S_{M1}$ = the MCER, 5 percent damped, spectral response acceleration parameter at a period of 1 s adjusted for site class effects as defined in Section 11.4.3

The MCER spectral response acceleration parameter for short periods ($S_{MS}$) and at 1 s ($S_{M1}$), adjusted for Site Class effects, shall be determined by Eqs. 11.4-1 and 11.4-2, respectively.

\begin{align*}
S_{MS} &= F_a S_s \\
S_{M1} &= F_v S_1
\end{align*}

Where:

$S_s$ = the mapped MCER spectral response acceleration parameter at short periods as determined in accordance with Section 11.4.1

$S_1$ = the mapped MCER spectral response acceleration parameter at a period of 1 s as determined in accordance with Section 11.4.1

and site coefficients $F_a$ and $F_v$ are defined in Tables 11.4-1 and 11.4-2, respectively.
Determination of Ground Motion Values and Spectra

- **Determination of Risk Category and Site Classification**
  This example illustrates the determination of seismic design parameters for a site in St. Louis, Missouri. The site is located at 38.60°N latitude, -90.2°W longitude.

  Using the results of a site-specific geotechnical investigation and the procedure specified in Standard Chapter 20, the site is classified as **Site Class B**.

- **Determination of Design Spectral Response Acceleration Coefficients**
  ASCE 7-10 Section 11.4.1 requires that spectral response acceleration parameters $S_s$ and $S_1$ be determined using the maps in Chapter 22, Figures 22-1 and 22-2.

  $S_s = 0.438g = 4.296 \text{ m/s}^2$

  $S_1 = 0.168g = 1.648 \text{ m/s}^2$

  Using these mapped spectral response acceleration values and the site class, site coefficients $F_a$ and $F_v$ are determined in accordance with Section 11.4.3, using Tables 11.4-1 and 11.4-2.

  Using Table 11.4-1, for $S_s \leq 0.25 => F_a = 1.0$ and for $S_s = 0.5 => F_a = 1.0$, for site class B.

  Using linear interpolation for $S_s = 0.438g => F_a = 1.0$

  Using Table 11.4-2, read $F_v = 1.0$ for $S_1 \leq 0.1$ and $F_v = 1.0$ for $S_1 = 0.2$ for Site Class B. Using linear interpolation for $S_1 = 0.168g => F_v = 1.0$
Using Equations 11.4-1 and 11.4-2 to determine the adjusted maximum considered earthquake spectral response acceleration parameters:

\[ S_{MS} = F_a S_S = 1.0 (0.438g) = 0.438g \]

\[ S_{M1} = F_v S_1 = 1.0 (0.168g) = 0.168g \]

Using Equations 11.4-3 and 11.4-4 to determine the design earthquake spectral response acceleration parameters:

\[ S_{DS} = \frac{2}{3} S_{MS} = \frac{2}{3} (0.438g) = 0.292g \]

\[ S_{D1} = \frac{2}{3} S_{M1} = \frac{2}{3} (0.168g) = 0.112g \]

Given the site location read ASCE 7-10 Figure 22-12 for the long-period transition period, \( T_L = 12 \text{ s} \).

The design spectrum is constructed in accordance with Provisions Section 11.4.5 using Provisions Figure 11.4-1 and Provisions Equations 11.4-5, 11.4-6 and 11.4-7. The design spectral response acceleration ordinates, \( S_a \), may be divided into four regions based on period, \( T \), as described below.

From \( T = 0 \) to \( T_0 = 0.2 \frac{S_{DS}}{S_{DS}} = 0.2 \frac{0.292g}{0.292g} = 0.076 \text{ s} \), \( S_a \) varies linearly from \( 0.4S_{DS} \) to \( S_{DS} \).

From \( T_0 \) to \( T_S = \frac{S_{D1}}{S_{DS}} = \frac{0.112g}{0.292g} = 0.383 \text{ s} \), \( S_a \) is constant at \( S_{DS} \).

From \( T_S \) to \( T_L \), \( S_a \) is inversely proportional to \( T \), being anchored to \( S_{D1} \) at \( T = 1 \text{ second} \).

At periods greater than \( T_L \), \( S_a \) is inversely proportional to the square of \( T \), being anchored to \( \frac{S_{D1}}{T_L} \) at \( T_L \).