8. Rotational stiffness

The rotational stiffness of a joint should be determined from the flexibilities of its basic components, each represented by an elastic coefficient according to EN 1993-1-8 6.3.2

Stiffness coefficients for basic joint components according to EN 1993-1-8, Table 6.11

**Column web panel in shear:**

Stiffened: \( k_1 = \infty \)

Unstiffened: \( k_1 = 0.38 \times \frac{A_{wc}}{\beta x_z} \)

\( z \) – distance to the first bolt row;

\( \beta \) – transformation parameter, see EN 1993-1-8 5.3.7

\( A_{wc} \) – shear area of the column

**Column web in compression:**

Stiffened: \( k_2 = \infty \)

Unstiffened: \( k_2 = 0.7 \times \frac{b_{eff,c,wc} x t_{wc}}{d_c} \)

\( b_{eff,c,wc} \) is specified in "Column web in transverse compression" 4.2.1

\( d_c \) – column straight portion of the web;

**Column web in tension:**

Stiffened welded connection: \( k_3 = \infty \)

Unstiffened or stiffened bolted connection or unstiffened welded connection:

\( k_3 = 0.7 \times \frac{b_{eff,tr,wc} x t_{wc}}{d_c} \)

\( b_{eff,tr,wc} \) is specified in “Column web in tension” 4.1.1

**Column flange in bending:**

\( k_4 = 0.9 \times l_{eff,k4} x \frac{t_f c}{m_L} \)

**End-plate in bending:**

\( k_5 = 0.9 \times l_{eff,k5} x \frac{t_p^3}{m^3} \)

**Bolts in tension (for a single bolt row):**

\( k_{10} = 1.6 \times \frac{A_s}{L_b} \)

**Concrete in compression (including grout):**

\( k_{13} = \frac{E_c}{1.275 \times E_s} \times \sqrt{\frac{b_{eff} x l_{eff}}{A_s}} \)
Base plate in bending under tension (for a single bolt row in tension)

- With prying forces:
  \[ k_{15} = 0.85x_{\text{eff}} \frac{t_p}{m^3} \]
- Without prying forces:
  \[ k_{15} = 0.425x_{\text{eff}} \frac{t_p}{m^3} \]

\( t_p \) - thickness of the base plate;

Anchor bolts in tension

- With prying forces:
  \[ k_{16} = 1.6x \frac{A_s}{L_b} \]
- Without prying forces:
  \[ k_{16} = 2x \frac{A_s}{L_b} \]

Total stiffness coefficient for one bolt row in tension is calculated taking into account the effective lengths for individual tensioned rows and the effective lengths for tensioned rows as part of a group according to EN 1993-1-8, 6.3.3.1(6.30)

\[ k_{\text{eff}} = \frac{1}{\sum_i k_i} \]

Note:

1) In the case of a beam-to-column joint with an end-plate connection (Moment End Plate joint), \( k_{\text{eff}} \) should be based upon the stiffness coefficients \( k_i \) for:
   - the column web in tension (\( k_3 \)),
   - the column flange in bending (\( k_4 \)),
   - the end-plate in bending (\( k_5 \));
   - the bolts in tension (\( k_{10} \)).

2) In the case of a beam splice with bolted end-plates (Apex joint), \( k_{\text{eff}} \) should be based upon the stiffness coefficients \( k_i \) for:
   - the end-plate in bending (\( k_5 \)),
   - the bolts in tension (\( k_{10} \)).

3) In the case of a Base plate joint \( k_{\text{eff}} \) should be based upon the stiffness coefficients \( k_i \) for:
   - concrete in compression (\( k_{13} \)),
   - base plate in bending under tension (\( k_{15} \)),
   - anchor bolts in tension (\( k_{16} \)).

Total stiffness coefficient for the tensioned area according to EN 1993-1-8, 6.3.3.1(6.29)

\[ k_{\text{eq}} = \frac{\sum k_{\text{eff}}xh_r}{z_{\text{eq}}} \]

\( h_r \) – distance between bolt-row and the centre of compression;
The equivalent lever arm according to EN 1993-1-8, 6.3.3.1 (6.31):

\[ z_{eq} = \frac{\sum k_{eff} x h_r^2}{\sum k_{eff} x h_r} \]

The initial rotational stiffness for a beam-to-column joint or beam splice is calculated according to EN 1993-1-8, 6.3.1(6.27):

\[ S_{j,ini} = Ex \frac{z_{eq}^2}{\sum k_i} \]

**Note:**

1) The rotational stiffness \( S_j \) of a column base subject to combined axial force and bending moment should be calculated according to EN 1993-1-8 Table 6.12. This method uses the following stiffness coefficients:

- \( k_{t,L} \) – tension stiffness coefficient of the left hand side of the joint and should be taken as equal to the sum of the stiffness coefficients \( k_{15} \) and \( k_{16} \) on the left hand side of the joint.
- \( k_{t,R} \) – tension stiffness coefficient of the right hand side of the joint and should be taken as equal to the sum of the stiffness coefficients \( k_{15} \) and \( k_{16} \) on the right hand side of the joint.
- \( k_{c,L} \) – compression stiffness coefficient of the left hand side of the joint and should be taken as equal to the stiffness coefficients \( k_{13} \) acting on the left hand side of the joint.
- \( k_{c,R} \) – compression stiffness coefficient of the right hand side of the joint and should be taken as equal to the stiffness coefficients \( k_{13} \) acting on the right hand side of the joint.

2) Rotational stiffness classification: A joint may be classified as rigid, nominally pinned or semi-rigid according to its rotational stiffness, by comparing its initial rotational stiffness \( S_{j,ini} \) with the classification boundaries given in EN 1993-1-8, 5.2.2.5
Rotational Stiffness Calculation:

The secant stiffness according to EN 1993-1-8, 5.1.2(4)

\[ S_j = \frac{S_{j,ini}}{\mu} \]

The stiffness ratio \( \mu \) should be determined from the following according to EN 1993-1-8 5.1.2(4):

\[ M_{j,Ed} \leq \frac{2}{3} x M_{j,Rd} \rightarrow \mu = 1 \]

\[ \frac{2}{3} x M_{j,Rd} < M_{j,Ed} \leq M_{j,Rd} \rightarrow \mu = \left( 1.5 \times \frac{M_{j,Ed}}{M_{j,Rd}} \right)^{\psi} \]

\( \psi \) – shape factor obtained from EN 1993-1-8 Table 6.8

The following rotation parameters are given in EN 1993-1-8 5.1.2 Figure 5.1.

Rotation at elastic limit:

\[ \phi_{el} = \frac{2}{3} x \frac{M_{j,Rd}}{S_{j,ini}} \]

Rotation at plastic limit:

\[ \phi_{pl} = \frac{M_{j,Rd}}{S_j} \]

Elastic – plastic rotational stiffness:

\[ S_{j,el-pl} = \frac{M_{j,Rd} - \frac{2}{3} x M_{j,Rd}}{\phi_{pl} - \phi_{el}} \]